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Algebra (Even and odd permutations)

Q.1. Define even and odd permutations and given an example.

Soln: Even and odd permutations:→

A permutation is said to be even or odd according as it can be expressed as a product of even or odd number of transpositions.

Let $P = \begin{pmatrix} 1 & 2 & 3 & \cdots & n \\ a_1 & a_2 & a_3 & \cdots & a_n \end{pmatrix}$ be a permutation of degree n. The pair (i, j) is said to be regular if $i-j$ and $a_i - a_j$ both have the same sign. Otherwise irregular. Thus, for irregularity of any pair (i, j), $(i-j)$ and $(a_i - a_j)$ are of opposite signs. The number of irregular pairs denotes number of inversions.

A permutation of a set of integers onto itself is even or odd according as it contains an even or odd number of inversions.

For example. I $\begin{pmatrix} 2 & 3 & 4 \\ 2 & 3 & 4 \end{pmatrix} \Rightarrow$ No inversion

II $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} \Rightarrow$ Two inversions.

(III) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 3 & 2 & 4 & 1 \end{pmatrix} \Rightarrow$ Eight inversions

which are Even permutations.

And IV $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} \Rightarrow$ Three inversions

V $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} \Rightarrow$ one inversion

which are odd permutations.

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Q.2- Prove that of the $\lfloor n$ permutations of n symbols, $\frac{\lfloor n}{2}$ are even permutation and $\frac{\lfloor n}{2}$ are odd permutations.

Proof: Let P_n be the set of all permutation on n distinct symbols. The P_n contains $\lfloor n$ distinct permutations of degree n. Also P_n is a group with respect to permutation multiplication as composition.

Out of these $\lfloor n$ permutations, let the even permutations be $E_1, E_2, E_3 \cdots E_p$ and the odd permutation be $O_1, O_2, O_3 \cdots O_q$ so that $p+q = \lfloor n$

Let $t \in P_n$ be arbitrary such that t is a transposition so that t is an odd permutation. Let t be operated on each of E_i $(i=1,2,3,\cdots p)$ and similarly on each O_i $(i=1,2,3,\cdots q)$.

Now, since $(P_n, *)$ is a group and therefore, $tE_i \in P_n$ and $tO_j \in P_n$ for $1 \le j \le p$, $1 \le j \le q$.

The permutations tE_i for $i=1,2,\cdots p$ are all odd permutation. Here, we see that transposition is an odd permutation and product of even and odd permutation is an odd permutation. Also, these permutations are all distinct.

For, $tE_i = tE_j \Rightarrow E_i = E_j$

$\therefore tE_i \neq tE_j$ if $E_i \neq E_j$

Since, a permutation can not be both even and odd. We have that the even permutations $E_1, E_2, E_3, \cdots E_p$ are equal to of even permutations $tO_1, tO_2, \cdots tO_q$.

Consequently, $p = q$. Also $p+q = \lfloor n$

$\Rightarrow p = q = \frac{\lfloor n}{2}$ Proved